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234. Proposed by G. B. M. ZERR, A. M., Ph. D., Parsons, W. Va.

Find the first negative pedal of an ellipse semi-axes a, b, referred to origin as center, and show that its entire area is $\frac{\pi}{2}\left[\frac{(a^2+b^2)^2}{4ab}+ab\right]$.

Solution by the PROPOSER.

$r\cos(\theta-\phi)=p=\frac{b}{\sqrt{(1-e^2\cos^2\phi)}}$, is the equation of a straight line perpendicular to the radius vector at its extremity. The envelope of this line is the first negative pedal. Differentiating we get

$$\sin(\theta-\phi)\sqrt{(1-e^2\cos^2\theta)}+\frac{e^2\cos(\theta-\phi)\sin\phi\cos\phi}{\sqrt{(1-e^2\cos^2\phi)}}=0.$$

$$\cos(\theta-\phi)=\frac{b}{r\sqrt{(1-e^2\cos^2\phi)}},\quad \sin(\theta-\phi)=\frac{\sqrt{[r^2(1-e^2\cos^2\phi)-b^2]}}{\sqrt{(1-e^2\cos^2\phi)}}.$$

These values from the equation of tho line substituted in the derived equation gives us

$$r^2=\frac{b^2}{(1--e^2\cos^2\phi)^2}-\frac{e^2b^2(1-e^2)\cos^2\phi}{(1-e^2\cos^2\phi)^3}$$

as the pedal sought.

$$\therefore A=2a^2b^2\int_0^{\frac{1}{2}\pi}\left[\frac{a^2}{(a^2\sin^2\phi+b^2\cos^2\phi)^2}-\frac{b^2(a^2-b^2)\cos^2\phi}{(a^2\sin^2\phi+b^2\cos^2\phi)^3}\right]d\theta.$$

$$\int_0^{\frac{1}{2}\pi}\frac{d\phi}{(a^2\sin^2\phi+b^2\cos^2\phi)^2}=\frac{\pi(a^2+b^2)}{4a^3b^3}.$$

Differentiating this with respect to b we get

$$\int_0^{\frac{1}{2}\pi}\frac{\cos^2\phi\,d\phi}{(a^2\sin^2\phi+b^2\cos^2\phi)^3}=\frac{\pi(3a^2+b^2)}{16a^3b^5}.$$

$$\therefore A=\frac{\pi a(a^2+b^2)}{2b}-\frac{\pi(a^2-b^2)(3a^2+b^2)}{8ab}=\frac{\pi}{8ab}(a^4+6a^2b^2+b^4)$$

$$=\frac{\pi}{2}\left[\frac{(a^2+b^2)^2}{4ab}+ab\right].$$

Also solved by J. Scheffer.